Abstract: Micro Aerial Vehicles have become subject of attention in different research areas, being autonomous stable flight one of the issues that has attracted special interest. Typically, a well known control strategy to achieve autonomous flight is the PID controller, in particular for quadcopters. However simple its implementation is, gain tuning is of paramount concern since a quadcopter’s motion is a nonlinear under-actuated system. To address this issue, simulations come in handy to test different control strategies aimed at finding an adequate set of gains. In this context, this paper presents a study where a non-linear PD controller is designed to control the stationary flight of a quadrotor with its implementation in simulation. Our contribution in this work is that of including in our model the use of stochastic noise injected to the references in the controller. Our goal is that of modelling a scenario where readings about the reference are perturbed with noise due to communication errors or unreliable global positioning system, which could greatly affect the performance of the PD controller. Our results indicate that the proposed approach is effective and robust under the presence of this noise.

Keywords: Micro Aerial Vehicles, quadrotor architecture, PD controller, design optimization, stochastic signals.

1. INTRODUCTION

This paper focuses on the design of a control technique for a non-linear and unstable system subjected to non-deterministic perturbations. Recently, researchers have shown an increased interest in designing control techniques for a particular type of UAV, the quadrotor. The latter is an under-actuated platform, but whose model is well understood and modelled in order to develop flight controllers.

One of the most significant current discussions in MAV’s is what control technique is used for hovering without stabilization problems. In this regard, most of the authors develop their solutions only in ideal environments, hence some factors are not taken into account such as air drag, ground effect, among other complications.

Motivated by the above, in this work we focus on a particular aspect that becomes relevant when designing a controller for the quadcopter. This aspect is that of modelling stochastic signals within the stabilization problem, specifically, when the vehicle has to reach a reference point whose values are subjected to perturbations. However, our hypothesis is that if such perturbations can be characterized as stochastic then such perturbations can be mitigated with an adequate modelling that takes into account such stochasticity.

The stochastic signals can be used to simulate noise or perturbations used to excite a dynamic system to obtain more realistic results compared to a deterministic signals. Therefore, in this work we present a no-linear modelling for a quadrotor based on a PID controller where a stochastic set point and stochastic disturbance are assumed. Thus, under a parametric optimization approach, we aimed at designing a PD controller for the flight of a quadrotor with such assumptions.

To achieve our goal, the approach presented in this work begins by obtaining the dynamic model of the quadrotor using the Newton-Euler formulation to identify the states to be controlled. The PD controller is designed to be tuned by parametric optimization using the gradient optimization algorithm, to stabilize the quadrotor in hover flight, which is subject to a stochastic signals that is generated to disturb the system.

* This work was supported by CONACYT.
Our results indicate that acceptable tracking of the model is obtained by our designed PD controller, showing better performances by reducing the scaling factor “R” despite the stochastic disturbances.

In order to describe our proposed approach, the rest of this paper has been organized as follows: section II presents the Related Work shown research on nonlinear controllers with some disturbances; section III describes the quadrotor model; section IV presents the methodology for the stochastic signal generation; section V describes the PD controller used in this work; section VI presents the results obtained in simulations; finally, the last section provides concluding remarks and some possible avenues for the future work.

2. RELATED WORK

Autonomous system of great acceptance in the industry and research are those of Unmanned Aerial Vehicles (UAV). For these vehicles, it has been developed a wide line of research on control techniques, which has enabled to achieve solutions to the problem of stability and attitude. The most common strategies are shown in Andrew Zulu (2014), such as, PID, LQR, adaptive control, fuzzy logic, neuronal networks, genetic control. However, because the dynamics of the quadrotor is complex and its mathematical models are nonlinear, nonlinear controllers have been developed, such as, Feedback linearization, Backstepping, Sliding Mode Control.

The standard theory of control requires the mathematical model of the dynamics of quadrotor. Several authors used Newton-Euler approach, some of them are Samir Bouabdallah (2007), J. Zhu (2015), Yaser Alothman (2015) and Ying Feng (2015), whose work has been used as a basis for the work presented in this paper.

Regarding the design of flight controller Keun Uk Lee (2012) addresses the PD controller for the attitude of a quadrotor, which shows an improvement in the pitch angle although a large error in the roll angle w.r.t. to the steady state.

Somasiri et al. (2015) demonstrated that effectiveness of a nonlinear geometric PID controller for attitude stabilization of a quadrotor, because, nonlinear PID is capable of quickly stabilizing any desired vertically upright configuration for large deviations from the desired set point.

Preliminary work on PID controller was undertaken by An et al. (2016) and design three channels to control the attitude of quadrotor, namely, roll, pitch and yaw channels. For each channels they use PID controllers. The goal of using adaptive algorithm is to make controller tune the PID parameters on-line according to the quadrotor changes. This controller requires no knowledge of the plant to be controlled and the self-tunning controllers are designed to tracking the signal for the roll, pitch and yaw respectively of a quadrotor.

A technique such as optimal control is applied by Reyad et al. (2016), where it is introduced the design of an optimal PID to improve attitude stabilization of an UAV, the proposed controller utilizes the differential evolution optimization algorithm to tune the parameters of the classical PID controller. They demonstrated superiority in rise time, settling time, overshoot and steady state error versus Ziegler-Nichols method, fuzzy tuner and genetic algorithm.

Preliminary work on stochastic disturbances was undertaken by Chen et al. (2009), who use the probability density function strategy for drive the PDF of the random tracking error for a class for robotic manipulator to approach the desired PDF, where the tracking error may be subjected to noises that follow non-Gaussian probability distributions, and he demonstrated the method can easily external to robot systems with a stochastic external uncertainties.

A similar pattern emerges in studies of Cai et al. (2011), where the aim of him paper is to determine a robot performance through ILC algorithm. This algorithm has been implemented for systems with stochastic disturbances, and he demonstrated that the performance obtained with the filters of his paper outperform heuristic filters previously proposed in the literature and he examined for two case when a preconditioning PID feedback controller has been applied and the ILC applied directly to the robot.

These findings suggest the use of stochastic signals in techniques of control, to prove the system performance, which would enable a better modelling of the quadrotor whose reference point values may be subjected to noise, either because the system that sets the reference point is under perturbations or because the the transmission of the data is slightly corrupted with noise.

3. QUADROTOR MODEL

On the whole, the mathematical model of a quadrotor is derived doing the following assumptions

- The quadrotor is considered as rigid body.
- The air drag is negligible.
- The body of quadrotor is symmetrical.

Two operations frames are introduced, the inertial frame $E$ and the body frame $B$. The first frame is fixed to the Earth, with gravity force in the negative $z$ direction, and the second is attached to the quadrotor center of mass (CM) and describes its orientation. These frames are depicted in Figure 1.

In the inertial frame $E$ the velocity and position is defined as $(u, v, w)$ and $(x, y, z)$. The Euler angles
Fig. 1. Frame system for modeling the quadrotor dynamics.

\((\phi, \theta, \psi)\) and the angular rates towards body axis \((p, q, r)\) are defined in the body frame.

The nonlinear model of a quadrotor is given in Bouabdallah et al. (2005). This one provides quadrotor’s full dynamic, and it is represented by the following system of ordinary differential equations

\[
\begin{align*}
m\ddot{X} &= (\sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi)U_1 \\
m\ddot{Y} &= (-\cos \phi \sin \theta + \sin \psi \sin \theta \cos \phi)U_1 \\
m\ddot{Z} &= (\cos \theta \cos \phi)U_1 - mg \\
I_x \ddot{\phi} &= (I_y - I_z)\dot{\psi} + U_2 - J_r \phi U_5 \\
I_y \ddot{\theta} &= (I_z - I_x)\dot{\psi} + U_3 - J_r \phi U_5 \\
I_z \ddot{\psi} &= (I_x - I_y)\dot{\theta} + U_4 \\
\end{align*}
\]

(1)

Where \(U_1, U_2, U_3, U_4, U_5\) are the system control inputs, these inputs corresponds to the change of the motors forces \((F_i)\), \(m\) is the quadrotor mass, \(I_x, I_y, I_z\) are the inertial matrix applied to the quadrotor center of mass (CM). \(l\) is the distance between the motor and the quadrotor CM. Finally, the inertia propeller of variable \(J_r\) is not taken into account since it is negligible \((J_r \approx 0)\)

Zhu et al. (2015) identifies eight space state equations to solve the problem of stationary flight of quadrotor; see (2). The resultant force denoted by \(\ddot{Z}\) is in the inertial frame \(E\), and the angular rates \((\phi, \theta, \psi)\) are in the body frame \(B\). The state space equations are given by

\[
\begin{align*}
\dot{z} &= w \\
\dot{w} &= \frac{1}{m}(\cos \theta \cos \phi)U_1 - g \\
\dot{\phi} &= (p \cos \theta + q \sin \theta \sin \phi + r \cos \phi \sin \theta) \frac{1}{\cos \theta} \\
\dot{\theta} &= q \cos \phi + r \sin \phi \\
\dot{\psi} &= (q \sin \phi + r \cos \phi) \frac{1}{\cos \theta} \\
\dot{p} &= \sqrt{2}(U_2 + qr(I_y - I_z))/I_x \\
\dot{q} &= \sqrt{2}(U_3 + pr(I_z - I_x))/I_y \\
\dot{r} &= \sqrt{2}(U_4 + pq(I_x - I_y))/I_z \\
\end{align*}
\]

(2)

In this work, we selected as quadrotor model the system given by (2).

Table 1 describes the quadrotor variables and the parameters are defined in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(z)</td>
<td>m</td>
<td>Height</td>
</tr>
<tr>
<td>(w)</td>
<td>(m/s)</td>
<td>z-axis velocity</td>
</tr>
<tr>
<td>(\phi)</td>
<td>rad</td>
<td>Roll angle</td>
</tr>
<tr>
<td>(\theta)</td>
<td>rad</td>
<td>Pitch angle</td>
</tr>
<tr>
<td>(\psi)</td>
<td>rad</td>
<td>Yaw angle</td>
</tr>
<tr>
<td>(p)</td>
<td>(rad/s)</td>
<td>Angular rate on x-axis</td>
</tr>
<tr>
<td>(q)</td>
<td>(rad/s)</td>
<td>Angular rate on y-axis</td>
</tr>
<tr>
<td>(r)</td>
<td>(rad/s)</td>
<td>Angular rate on z-axis</td>
</tr>
<tr>
<td>(U_1)</td>
<td>N</td>
<td>Control Input on height</td>
</tr>
<tr>
<td>(U_2)</td>
<td>N</td>
<td>Control Input on roll</td>
</tr>
<tr>
<td>(U_3)</td>
<td>N</td>
<td>Control Input on pitch</td>
</tr>
<tr>
<td>(U_4)</td>
<td>N (*m)</td>
<td>Control Input on yaw</td>
</tr>
</tbody>
</table>

Table 2. Parameter Description

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>1kg</td>
<td>Quadrotor mass</td>
</tr>
<tr>
<td>(g)</td>
<td>9.81m/s(^2)</td>
<td>Gravity acceleration</td>
</tr>
<tr>
<td>(I_{x,y,z})</td>
<td>0.08kg (*m^2)</td>
<td>Body inertia</td>
</tr>
<tr>
<td>(l)</td>
<td>0.24m</td>
<td>Distance between motor and CM</td>
</tr>
</tbody>
</table>

4. STOCHASTIC SIGNALS

Step inputs are quite useful as test signals in classical compensation of linear systems, since performance specifications, such as overshoot, settling time and rise time, are given in terms of step response characteristics. Besides, it is known that if the linear system has a good response, it will have a satisfactory response to a general input.

On the other hand, stochastic test signals overcome the limitations imposed by the step. They are more realistic because usually systems operate with no deterministic signals. In addition, we can module the amplitude and frequency bandwidth of them. This approach, permit us to simulate scenarios closer to reality, we can consider nonlinear models.

The model for generating stochastic signals is the one proposed in (Hasdorff, 1975), shown in Figure 2.
This model has, in the first place, a (pseudo) random number generator, which is followed by a hold whose output feeds the input of the linear filter \( w(t) \) with transfer function \( W(s) \).

The test stochastic signal, \( r(t) \), can be selected as desired by controlling the probability function of the random number generator and by controlling \( W(s) \).

![Block diagram of stochastic test signal generator](image)

To get the filtering simple, first and second order filters were chosen as follows:

\[
W_1(s) = \frac{a}{s + a} \quad (3)
\]

\[
W_2(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4)
\]

where \( W_1(s) \) gives a corner frequency of \( a \) and \( W_2(s) \) has a corner frequency \( \omega_n \) and \( \zeta \) is the damping factor.

The variance of the test signal \( r(t) \) using \( W_1(s) \) and \( W_2(s) \) is given by

\[
\sigma_r^2 = \frac{a}{2} \sigma_{RN} h \quad (5)
\]

where \( \sigma_{RN} \) is the variance of the numbers obtained by the random number generator, and \( h \) is the period of the random number generator. The equation (5) has already been proved in Newton and Kaiser (1957).

4.1 Choice cost function with stochastic inputs

The cost function is selected how (Hasdorff, 1975)

\[
J = \int_{t_0}^{t_f} e(t)^2 dt = \int_{t_0}^{t_f} (y_{sp}(t) - y(t))^2 dt \quad (6)
\]

where \( y(t) \) is the output of the plant and \( y_{sp}(t) \) is the set point of the plant. The above equation is equivalent to minimizing the following

\[
J = \frac{1}{t_f - t_0} \int_{t_0}^{t_f} e^2(t) dt \quad (7)
\]

which is the mean square value error. In addition, when dealing with stationary ergodic process we have

\[
\lim_{t_f - t_0 \to \infty} J = E[e^2] \quad (8)
\]

where \( E[e^2] \) is the expected value of \( e^2 \).

From (8) we have that minimizing the criterion (6) is equivalent to minimizing the ensemble average of the squared error when the simulation interval \([t_0, t_f]\) is long enough. So, it is sufficient to select one test stochastic signal from the generator (Figure 2) to optimize system response for all signals from it.

5. PD CONTROLLER

The PD control inputs of the quadrotor system (2) are given by

\[
U_1 = k_{p1}(z_{sp} - z(t)) - k_{d1}\dot{z} \\
U_2 = k_{p2}(\phi_{sp} - \phi(t)) - k_{d2}\dot{\phi} \\
U_3 = k_{p3}(\theta_{sp} - \theta(t)) - k_{d3}\dot{\theta} \\
U_4 = k_{p4}(\psi_{sp} - \psi(t)) - k_{d4}\dot{\psi}
\]

where \( z_{sp}, \phi_{sp}, \theta_{sp}, \psi_{sp} \) are the set points. The parameters \( k_{pi} \) and \( k_{di} (i = 1, \ldots, 4) \) are found by means of gradient conjugate algorithm with the following cost function

\[
J(u) = \int_{t_0}^{t_f} [(z_{sp} - z(t))^2 + (\phi_{sp} - \phi(t))^2 + (\theta_{sp} - \theta(t))^2 + (\psi_{sp} - \psi(t))^2 + R \left( \sum_{i=1}^{4} k_{pi}^2 + \sum_{i=1}^{4} k_{di}^2 \right)] dt
\]

where \( R > 0 \) is a weighting factor to control the magnitude of the control inputs.

By substituting (9) into (2), and to handle (10) the equations of the quadrotor model (2) are augmented by the new state variable

\[
\dot{J} = (z_{sp} - z(t))^2 + (\phi_{sp} - \phi(t))^2 + (\theta_{sp} - \theta(t))^2 + (\psi_{sp} - \psi(t))^2 + R \left( \sum_{i=1}^{4} k_{pi}^2 + \sum_{i=1}^{4} k_{di}^2 \right)
\]

The state space augmented variable is given by \( x = [z, w, \phi, \theta, \psi, p, q, r, J(u)]^T \)

6. IMPLEMENTATION

To design the PD quadrotor controller using stochastic test signals, we consider a scenario where readings about the reference are perturbed with noise due to communication errors or unreliable global positioning system, the state variables roll (\( \phi \)), pitch(\( \theta \)), and yaw(\( \psi \)) are also perturbed.

A simple first order smoothing filter was used to generate the disturbance signals \( d_1(t) \), \( d_2(t) \) and \( d_3(t) \) of the state variables roll (\( \phi \)), pitch(\( \theta \)), and yaw(\( \psi \)); see (3) with \( a = 1 \).

\[
W(s) = \frac{1}{s + 1} \quad (12)
\]

Then, we have a cut-off frequency of \( \omega = 1 \) that implies the disturbance spectrum is principally flat.
in the passband of the signal exciting the quadrotor system.

Here we present the model equations that have the disturbance signals, this disturbance were added to simulate the noise or disturbances of the environment that can occasion the loss of the attitude of the quadrotor.

\[
\begin{align*}
\dot{\phi} &= (p \cos \theta + q \sin \theta \sin \phi + r \cos \phi \sin \theta) + d_1(t) \\
\dot{\theta} &= q \cos \phi + r \sin \phi + d_2(t) \\
\dot{\psi} &= (q \sin \phi + r \cos \phi) \frac{1}{\cos \theta} + d_3(t)
\end{align*}
\] (13)

In the case of the reference variable \( z_{sp} \), we used a second order filter (equation (4)) given by

\[
W(s) = \frac{17.47}{s^2 + 45.98s + 17.47}
\] (14)

to generate the disturbance signal \( d_4(t) \) and calculate \( z_{sp} + d_4(t) \). From the poles of the quadrotor system, we obtain the damping factor \( \zeta = 5.5 \) and the natural frequency of \( w_n = 4.18 \text{rad/sec} \).

By means of the poles of the system and employing the absolute stability region of the Runge-Kutta 4 method, we selected the integration step \( h \); see Lambert (1973).

In both cases, we used \( h = 0.04 \) as the period of the random number generator and a Gaussian distribution with mean of zero. The variance was \( \sigma^2_{RN} = 10 \) in the case of the reference variable, and for the disturbance inputs of the state variables, the variances are shown in Table 3.

Table 3. Parameters of random number generator

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1(t) )</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>( d_2(t) )</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>( d_3(t) )</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

We used a Polak-Ribière conjugate gradient method to solved the optimization problem (Nielsen, 2010), the parameters to be optimized are:

\[
u = [k_{p1}, k_{p2}, k_{p3}, k_{p4}, k_{d1}, k_{d2}, k_{d3}, k_{d4}]^T
\] (15)

The initial guess of the parameters is \( k_{pi} = k_{di} = 2.0 \) \( (i = 1, \ldots, 4) \), and the simulation interval is \( [0, 50] \).

The initial conditions of the quadrotor model are \( x(t_0) = (0, 0, \pi/4, \pi/4, \pi/4, 0, 0, 0) \), the set points are \( \phi_{sp} = 0, \theta_{sp} = 0, \psi_{sp} = \pi/6, z_{sp} = 5 \). To solve the ordinary differential equations (2) we chose a fourth order Runge-Kutta method with fixed step \( \Delta t = 0.04 \).

7. SIMULATION AND RESULTS

The nonlinear quadrotor model (2) augmented with (11) were code in Matlab, and we used the Polak-Ribière conjugate gradient method from (Nielsen, 2010). The model parameters are provided in Table 2 and the values obtained for \( k_{pi} \) and \( k_{di} \) \( (i = 1, \ldots, 4) \) are shown in Table 4. Table 5 shows the value of the cost function at \( u^* \) in (see (15)) where

\[ u^* = \arg \min J(u) \]

Figure 3 and Figure 4 shows the performance of the optimized PD controller with stochastic test signals, when using \( R = 10^{-5} \). The height \( z \) is close to stochastic reference signal \( z_{sp} + d_4(t) \). The other variables roll, pitch and yaw reach their set-points with brief oscillations.

Figure 5 shows the relative error of \( z \) respect to the stochastic set-point in percentage. Approximately a small error rate between 0% and 8% is observed.

The control inputs behavior is depicted in Figure 6, we can see the magnitude obtained with smaller values of \( R = 10^{-5} \).

Figure 7 shows the PD control performance when the period of the random number generator is diminishing to 0.01. In this case we must take the integration step also as 0.01. We can observe that there is a delay more marked between \( z \) and the setpoint.

From the numerical simulations, we can say that the controller has a acceptable behavior.

The dynamic stability of the quadrotor in closed-loop is given through the system poles, which are depicted in Figure 8. We can observe that each pole has a real part negative.

Table 4. Feedback Gains for various Values

<table>
<thead>
<tr>
<th>Gains</th>
<th>( R = 10^{-5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{p1} )</td>
<td>75.18</td>
</tr>
<tr>
<td>( k_{p2} )</td>
<td>7.660</td>
</tr>
<tr>
<td>( k_{p3} )</td>
<td>6.864</td>
</tr>
<tr>
<td>( k_{p4} )</td>
<td>3.644</td>
</tr>
<tr>
<td>( k_{d1} )</td>
<td>6.437</td>
</tr>
<tr>
<td>( k_{d2} )</td>
<td>10.068</td>
</tr>
<tr>
<td>( k_{d3} )</td>
<td>2.134</td>
</tr>
<tr>
<td>( k_{d4} )</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Table 5. Function Cost Values

<table>
<thead>
<tr>
<th>Function Cost</th>
<th>( J(u^*) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R = 10^{-5} )</td>
<td>8.829</td>
</tr>
</tbody>
</table>

8. CONCLUSIONS

In this work, we have presented the design and implementation of a PD controller for a non-linear
quadrotor model that is subject to stochastic perturbations. The purpose of the current study was to determine the performance of the controller with a set point subject to disturbance for a non-linear system. For the latter, our approach is based on the use of parametric optimization.

Our results suggest that in general, the proposed controller is acceptable even under scaling factor \( R \) changes. These changes indicate the weight factor given to the variables of the cost function. That is, decreasing the scaling factor increases the weight factor of the state variables. These results also indicate that, albeit within a slightly time lag, the controller effectively follows the set point subjected to stochastic disturbance. In the same way, the attitude states remain relatively stable when the desired set point is reached.

This research will serve as a basis for future studies on control strategies using stochastic signals as perturbations for non-linear and under-actuated systems. In our literature review, we noted that work on stochastic signals used to simulate disturbances in MAV’s was missing, most of the research work assumes ideal conditions, hence the relevance of our work.

The current study has only examined the control for the stationary flight of a quadrotor, not for translational flight. If we desire to control the quadrotor on the cartesian plane in terms of the translation component, it is required to add the ODE’s of the states \( x, y \) into equation 2 and define a new cost function. On the other hand, we added the disturbances in the Euler angles of the quadrotor to verify the adequate functioning of the attitude control.

In our future work, we will research the design of a controller for the translational flight to get a full control and navigation system.
Fig. 7. States variables behavior with tuned PD controller, and \( R = 10^{-5} \) with \( h = 0.01 \)

Fig. 8. Location of the poles, which guarantees the stability of the closed-loop system.

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loads control.


*The code developed in this work is available upon request, please write an email to: grodrig@inaoep.mx or carranza@inaoep.mx